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On models for turbulence modulation in fluid–particle flows

Clayton T. Crowe

School of Mechanical and Materials Engineering, Washington State University, Pullman, WA 99164-2920, USA

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Abstract

A model is introduced for the carrier phase turbulence in a fluid–particle flow based on the volume-averaged equations for the kinetic energy of the carrier phase. The model shows the trends observed by experiment, that is, small particles attenuate turbulence while large particles augment turbulence. The change in turbulence intensity is correlated with the particle loading and the ratio of the particle diameter to the turbulence length scale. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Fluid–particle flows; Turbulence; Turbulence modulation

1. Introduction

Turbulence is a key element in the flow of fluid–particle mixtures. It is responsible for the mixing of chemical species, the transfer of heat and the shear stresses (Reynolds stress) in the continuous phase. The presence of particles or a second phase on the turbulence of the continuous phase is known as turbulence modulation. There have been numerous studies, both numerical and experimental, on turbulence modulation. Several sources of turbulence in the carrier phase due to particles or droplets have been identified; stream line distortion due to the presence of the particles, the wake generated by particles, the modification of the velocity gradients in the carrier phase and the associated change in turbulence generation and the damping of turbulence motion by the drag force on the particles. Still there is no generally accepted model which is applicable to all flow conditions.

E-mail address: crowe@mme.wsu.edu (C.T. Crowe).

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A summary of turbulence modulation effects up to 1989 was published by Gore and Crowe. These data included both jets, flow in pipes, gas-particle and gas–liquid flows. The data showed a general trend; small particles tend to attenuate turbulence, large particles tend to enhance turbulence. The data appeared to correlate with the ratio of the particle size the turbulence length scale. Several experiments have been reported on turbulence modulation since 1989. Recent experiments by Hosokawa et al. (1998) with gas-particle flow in a vertical pipe show that the carrier phase turbulence is augmented at the pipe center but attenuated near the wall. Similar experiments by Varaksin et al. (1998) show the turbulence is attenuated uniformly across the pipe. Another vertical pipe flow experiment by Savolainen and Karvinen (1998) indicate turbulence is attenuated at high velocities and augmented at low velocities. Sato et al. (1996) have measured the modulation effects in particle laden jets and Kulick et al. (1994) report measurements in fully developed vertical channel flow. The same plot used by Gore and Crowe (1989) with the more recent data is shown in Fig. 1. One notes the same trends. A general model to quantitatively explain these trends is needed.

2. Turbulence modulation models

Several physical models have been proposed to explain the observed trends. Yuan and Michaelides (1992) proposed a model in which the velocity defect in the wake is responsible for

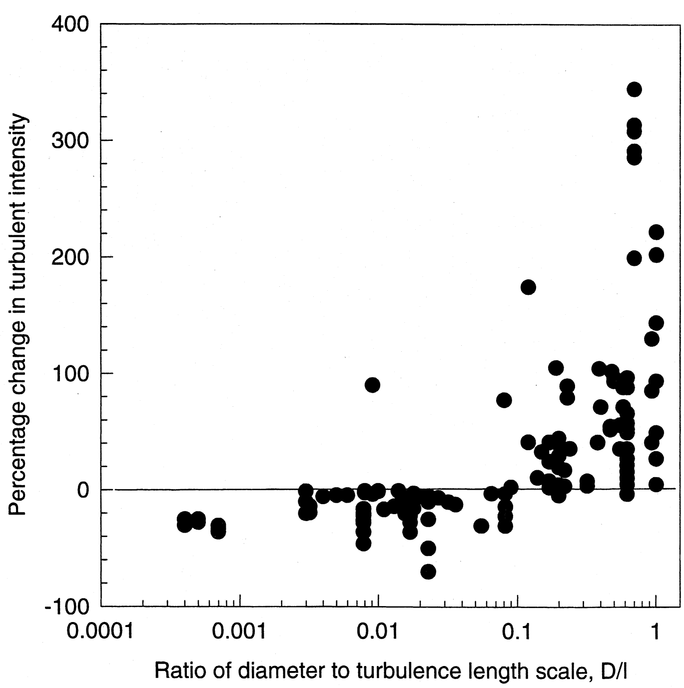

Fig. 1. Compilation of data on the effect of the dispersed phase on the turbulence of the carrier phase as a function of the particle size — turbulence length scale ratio.

the augmentation of turbulence and the work associated with the motion of the particulate phase is responsible for the attenuation of turbulence. They suggest that the turbulence generation is given by

$$G_k = D^2 \rho_c f(l_w)(u^2 - v^2) \tag{1}$$

where D is the particle diameter, ρ_c is the density of the carrier phase, $f(l_w)$ is a function of the wake size and u and v are the fluid and particle velocities, respectively. The above model should be modified to yield a positive value independent of the relative magnitudes of u and v and should also account for the interaction of particles and wakes as the particle concentration is increased. Yuan and Michaelides do report, however, good agreement with experimental results.

Yarin and Hetsroni (1994) utilize an idea similar to Yuan and Michaleides but employ a more detailed description of the wake. They predict that

$$\frac{\sqrt{u'^2}}{|v - u|} = \left(C \frac{\rho_c}{\rho_d} C_d^{3/2} \right)^{4/9} \tag{2}$$

where C_d is the drag coefficient and C is an empirical constant. This model appears to correlate data for turbulence generated solely by particles at very low particle volume fractions.

More recently, Kenning and Crowe (1997) have proposed a model for turbulence modulation for gas particle flows based on the work done by particle drag and the dissipation based on a length scale corresponding to the interparticle spacing. They stated that the generation rate of turbulence energy per unit mass of mixture is given by

$$G_k = \frac{f}{\tau_V} C (u - v)^2 \tag{3}$$

where f is the ratio of the drag coefficient to Stokes drag, C is the ratio of mass of the dispersed phase to the carrier phase in a two-phase mixture and τ_V is the particle response time. The corresponding dissipation was modeled

$$\epsilon = \frac{k^{3/2}}{\ell_h} \tag{4}$$

where ℓ_h is a hybrid length scale which approaches the interparticle spacing for interparticle spacings less than the dissipation length scale. This model was applied to gas-particle flow in a vertical tube and showed good agreement with experimental results obtained up to 1989 but has not been extended to the more recent data.

The majority of models for turbulence in fluid–particle flows are extensions of the $k - \varepsilon$ formulation used for single phase flows. The momentum equation for a fluid–particle mixture has an additional term that accounts for the fluid-dynamic force acting on the fluid per unit volume of mixture and is expressed as (Crowe et al., 1997)

$$f_{d,i} = \frac{1}{V} \sum_{i=1}^{N} 3\pi \mu_c D_i f_i (v_i - u_i) \tag{5}$$

where N is the number of particles in an averaging volume V, μ_c is the viscosity of the carrier phase, D_i is the particle diameter, f_i is the ratio of the drag coefficient to Stokes drag and v_i and u_i are the particle and fluid velocities, respectively. If the particle size is uniform and f_i is constant, the force term becomes

$$f_{d,i} = n3\pi\mu_c Df\frac{1}{N}\sum_{i=1}^{N}(v_i - u_i) = n3\pi\mu_c Df(\bar{v}_i - \bar{u}_i) \tag{6}$$

where n is the particle number density and the bar over the velocities signifies the number average. For uniform size particles, the number average particle velocity is equal to the volume average, $\langle v_i \rangle = \bar{v}_i$. The volume average of the carrier fluid velocity is also a reasonable approximation of the number average of the velocity at the particle positions. Finally, the force term can be expressed as

$$f_{d,i} = \bar{\rho}_d \frac{f}{\tau_V}(\langle v_i \rangle - \langle u_i \rangle) = \beta_V(\langle v_i \rangle - \langle u_i \rangle) \tag{7}$$

where $\bar{\rho}_d$ is the bulk density of the dispersed phase. There are other forms for this term that use time-averaged velocities or ensemble-averaged velocities. The main point is that the term is the drag force *per unit volume of mixture* and cannot be regarded as a force at a point in the flow. It is common practice in developing models for turbulence modulation to write the momentum equation for the carrier phase as

$$\rho_c\frac{\partial u_i}{\partial t} + \rho_c u_j\frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} + \rho_c g_i + \beta_V(v_i - u_i) \tag{8}$$

and treat the properties as values at a point. However, the properties are averaged properties, not local. The momentum equation that applies to a point in the carrier fluid is the Navier–Stokes equation which does not include a momentum coupling term. The procedure (Chen and Wood, 1985; Desjonqueres, 1987; Berlemont et al., 1990; Varaksin et al., 1998) is to first multiply Eq. (6) by the velocity to obtain a mechanical energy equation, express the velocities as an average value plus a deviation, take a temporal average and subtract the averaged mechanical energy equation to yield an equation for the turbulence kinetic energy. In general the form of the equation is

$$\rho\frac{\partial k}{\partial t} + \rho u_i\frac{\partial k}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\sigma\frac{\partial k}{\partial x_i}\right) - \rho\overline{u_i'u_j'}\frac{\partial u_i}{\partial x_j} + \beta_V\left(\overline{u_i'v_i'} - \overline{u_i'u_i'}\right) - \rho\varepsilon \tag{9}$$

One can readily identify the diffusion, generation and dissipation terms common to the single phase kinetic energy equation. The additional term accounts for the presence of the dispersed phase.

The problem with this formulation can be shown by applying it to a simple example. Suppose there was a flow in which the particles were artificially fixed in position as shown in Fig. 2. Assume the flow is steady and uniform (no temporal or spatial gradients in the averaged properties). This cannot be done experimentally without providing support for the particles but represents an ideal case (like a reversible process in thermodynamics) which can be used to check the models. A series of screens in the flow would approximate this flow configuration. In this case, one would expect that the turbulence generated by the fixed particles would be dissipated by viscous forces. Applying the above equation to this flow configuration; that is, setting $\partial/\partial t$ and $\partial/\partial x_i$ equal to zero, one finds

$$-\beta_V \overline{u_i' u_i'} - \rho\varepsilon = 0 \tag{10}$$

which is obviously incorrect since the dissipation ε is always positive. The problem lies in taking averaged values as being local values and treating the equations as if they represented a single phase flow with a local coupling term.

Another approach (Gillandt and Crowe, 1998) in deriving the turbulence energy equation for a fluid–particle mixture is to use the mechanical energy equation for the fluid phase of an incompressible fluid, namely,

$$\rho_c \frac{D}{Dt}\left(\frac{u_i u_i}{2}\right) = -\frac{\partial}{\partial x_i}(u_i p) + u_j \frac{\partial}{\partial x_j}\tau_{ij} + \rho_c u_i g_i \tag{11}$$

The velocities, pressure and shear stress are expressed as the sum of the volume-averaged quantities and the deviation from these values,

$$u_i = \langle u_i \rangle + \delta u_i,\ p = \langle p \rangle + \delta p,\ \tau_{ij} = \langle \tau_{ij} \rangle + \delta \tau_{ij} \tag{12}$$

These values are substituted in the mechanical energy equation and the resulting equation is volume averaged. The product of the volume averaged velocity and momentum equations is subtracted to yield an equation for the turbulence kinetic energy. The resulting equation is

$$\frac{\partial}{\partial t}(\alpha_c \rho_c k_c) + \frac{\partial}{\partial x_j}(\alpha_c \rho_c \langle u_j \rangle k_c) = \frac{\partial}{\partial x_j}\left(\sigma_k \frac{\partial k_c}{\partial x_j}\right) \quad \text{diffusion of } k_c$$

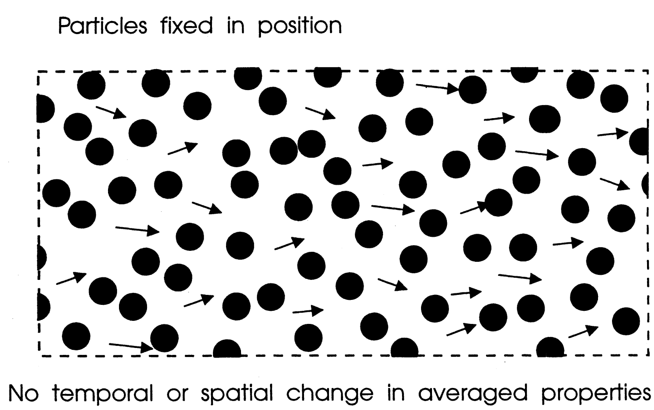

Fig. 2. Schematic of the flow configuration in which particles are fixed in position and fluid moves through the particle cloud.

$$-\alpha_c\rho_c\langle\delta u_i\delta u_j\rangle\frac{\partial}{\partial x_j}\langle u_i\rangle \quad \text{generation by gradients}$$

$$+\beta_V|\langle u_i\rangle-\langle v_i\rangle|^2 \quad \text{generation by particle drag}$$

$$+\beta_V(\langle\delta v_i\delta v_i\rangle-\langle\delta u_i\delta v_i\rangle) \quad \text{redistribution}$$

$$-\alpha_c\varepsilon \quad \text{dissipation} \tag{13}$$

One identifies two terms associated with the presence of the particle phase. One term reflects the conversion of mechanical work by the drag force into turbulence kinetic energy and the other, the redistribution term, represents the transfer of kinetic energy of the particle motion to kinetic energy of the carrier fluid. This term has been identified previously as four-way coupling (Elghobashi, 1994). In dilute flows it is usually small compared to the particle drag generation term but becomes important in dense phase flows when particle–particle collisions are a source of particle kinetic energy. Similar equation for turbulence kinetic energy have been developed by Kataoka and Serizawa (1989), Hwang and Shen (1993), Liljegren (1997).

Applying this equation to the idealized model where the particles are artificially fixed in position in the flow yields the following result,

$$\beta_V|\langle u_i\rangle|^2-\alpha_c\varepsilon=0 \tag{14}$$

which states that the turbulence energy produced by the drag forces is dissipated by viscous effects.

The dissipation term is expressed as

$$\varepsilon=\mu_c\langle\frac{\partial\delta u_i}{\partial x_j}\frac{\partial\delta u_i}{\partial x_j}\rangle \tag{15}$$

The presence of the particles provides surfaces that can support a stress. Thus the Kolmogorov length scale which represents the smallest scale in a single phase flow is no longer appropriate for a fluid–particle flow. The interparticle spacing and particle diameter become additional length scales that must be considered. Direct numerical simulations which treat the particles as point forces do not resolve the velocity gradients imposed by the particle surfaces so provide an incomplete simulation of the flow and evaluation of the dissipation term.

For a fully developed dilute fluid–particle flow in a vertical pipe the turbulence energy equation for the flow near the pipe center reduces to

$$-\alpha_c\rho_c\langle\delta u_i\delta u_j\rangle\frac{\partial\langle u_i\rangle}{\partial x_j}+\beta_V|\langle u_i\rangle-\langle v_i\rangle|^2-\alpha_c\varepsilon=0 \tag{16}$$

where the redistribution term has been neglected because it is small compared to the generation term due to particle drag and the diffusion term will not be important near the pipe centerline. Modeling the production term due to velocity gradients can be modeled as

$$-\alpha_c \rho_c \langle \delta u_i \delta u_j \rangle \frac{\partial \langle u_i \rangle}{\partial x_j} \simeq \alpha_c \rho_c \frac{k_0^{3/2}}{l} \tag{17}$$

where k_0 is the turbulence kinetic energy in the particle free flow and l dissipation length scale. The dissipation term is modeled as

$$\alpha_c \varepsilon \simeq \alpha_c \rho_c \frac{k^{3/2}}{l_h} \tag{18}$$

where l_h is the 'hybrid' length scale approximated as

$$\frac{1}{l_h} = \frac{1}{\lambda} + \frac{1}{l} \tag{19}$$

where λ is the interparticle spacing. Taking the particle–fluid velocity difference as the terminal velocity, $g\tau_V/f$ Eq. (14) can be rewritten to yield the turbulence modulation

$$\left(\frac{k}{k_0}\right)^{3/2} = \frac{1 + C\left(gL/U^2\right)^2 \rho_d UL/\mu_c (l/L)^3 1/18 f \sigma_0^3 (D/l)^2}{1 + \alpha_d^{1/3}(l/D)} \tag{20}$$

where L is the pipe diameter, U is the fluid velocity and σ_0 is the turbulence intensity of the particle-free flow. This equation shows that the modulation depends on the parameter identified by Gore and Crowe; namely, particle size-turbulence length scale ratio, D/l. The drag factor f is a function of the relative Reynolds number and can be approximated as

$$f^{5/2} = 0.058 \frac{g\tau_V D \rho_c}{\mu_c} \tag{21}$$

for particles falling at their terminal velocity. Also the volume fraction of the dispersed phase for $\alpha_d \ll 1$ can be expressed as

$$\alpha_d = \left(\frac{\rho_c}{\rho_d}\right) C \tag{22}$$

Substituting the expressions for f and α_d into Eq. (20) yields the following equation for turbulence modulation.

$$\left(\frac{k}{k_0}\right)^{3/2} = \frac{1 + 0.55C\left(gL/U^2\right)^{1.6} (\rho_c UL/\mu_c)^{0.2} (l/L)^{1.8} \sigma_0^{-3} (D/l)^{0.8}}{1 + (\rho_c/\rho_d)^{1/3} C^{1/3} (D/l)^{-1}} \tag{23}$$

The modulation of turbulence as predicted by Eq.(23) for a 10 m/s air flow with glass particles in a 10-cm pipe and a particle free turbulence intensity of 0.06 is shown in Fig. 3. The ratio of the turbulence length scale to the pipe diameter was taken as 0.1 (Hutchinson et al., 1971). The different curves correspond to different mass concentrations of 0.1, 1 and 5. For mass concentration of 0.1, the effect of the particles on the carrier-phase turbulence is minimal while the effect is pronounced for mass concentrations of 5. The data on this figure are for particles

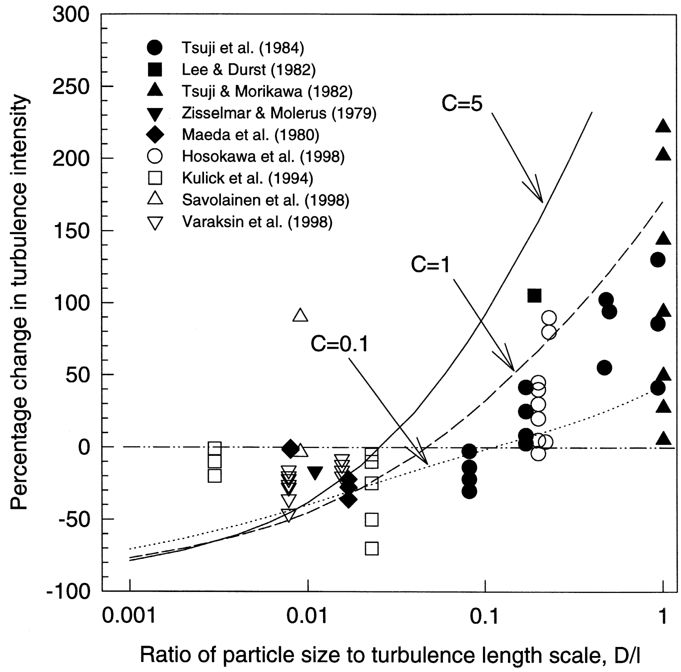

Fig. 3. Comparison of model predictions and data for the carrier-phase turbulence at the pipe centerline for fluid–particle flow in a vertical pipe.

in vertical pipe flows. The curves show the same trends as the data which lends credence to the model for turbulence modulation.

3. Conclusion

Models for turbulence modulation based on treating the carrier phase momentum equation as if it were a single phase flow equation with properties defined at a point leads to an incorrect result for a simplified flow configuration. One viable approach is to start with the equation for the mechanical energy of the carrier phase and perform the averaging procedures. This approach yields an equation which reduces to a reasonable result for the simplified flow and provides a correlation which is supported by trends observed in experimental results.

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